ΔΕΥΤΕΡΟ ΚΑΘΕΤΟ ΓΙΑ ΤΥΧΑΙΑ ΠΑΡΑΜΕΤΡΟ

$$\vec{b}(t) = \frac{c'(t)\times c''(t)}{\|c'(t)\times c''(t)\|}, \quad k(t)>0 \text{ και } c \text{ κανονική}$$

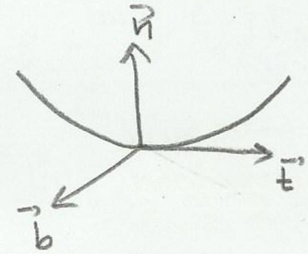
Απόδειξη

$$c' = \frac{dc}{dt} = \frac{dc}{ds}\cdot\frac{ds}{dt} = \mathring{c}\cdot\frac{ds}{dt} \Rightarrow \boxed{c' = \mathring{c}\,\frac{ds}{dt}}$$

$$c'' = \frac{dc'}{dt} = \frac{d}{dt}\left(\mathring{c}\,\frac{ds}{dt}\right) = \frac{d}{dt}(\mathring{c})\cdot\frac{ds}{dt} + \mathring{c}\cdot\frac{d^2s}{dt^2} =$$

$$= \frac{d}{ds}\left(\frac{dc}{ds}\right)\frac{ds}{dt}\cdot\frac{ds}{dt} + \mathring{c}\,\frac{d^2s}{dt^2} \Rightarrow$$

$$\Rightarrow \boxed{c'' = \mathring{\mathring{c}}\cdot\left(\frac{ds}{dt}\right)^2 + \mathring{c}\cdot\frac{d^2s}{dt^2}}$$



$$S = S(t) = \int_0^t \|c'(\sigma)\|\,d\sigma \Rightarrow \frac{ds}{dt} = \|c'(t)\|$$

$$c'\times c'' = \left(\mathring{c}\,\frac{ds}{dt}\right)\times\left(\mathring{\mathring{c}}\left(\frac{ds}{dt}\right)^2 + \mathring{c}\cdot\frac{d^2s}{dt^2}\right) =$$

$$= (\mathring{c}\times\mathring{\mathring{c}})\left(\frac{ds}{dt}\right)^3 = \left(\vec{t}\times\mathring{\vec{t}}\right)\left(\frac{ds}{dt}\right)^3 =$$

$$= (\vec{t}\times k\cdot\vec{n})\left(\frac{ds}{dt}\right)^3 = k\,\vec{b}\cdot\left(\frac{ds}{dt}\right)^3 = k\cdot\vec{b}\cdot\|c'(t)\|^3.$$

$$\|c'\times c''\| = |k|\cdot\|c'(t)\|^3$$

Τοτε, $\dfrac{c'\times c''}{\|c'\times c''\|} \overset{k>0}{=} \dfrac{k\cdot\vec{b}\cdot\cancel{\|c'(t)\|^3}}{|k|\cdot\cancel{\|c'(t)\|^3}} = \vec{b}$